### Mathematics of Finance

#### Changing a Fraction to a Percent
Express \( \frac{3}{40} \) as a percent.

| \( \frac{3}{40} \) | 0.075 | 7.5% |

To express a fraction as a percent, divide the numerator by the denominator, multiply by 100 to move the decimal point in the result two places to the right, and add a percent sign.

#### Changing a Decimal to a Percent
Express \( 0.96 \) as a percent

To express a decimal as a percent, multiply by 100 to move the decimal point two places to the right, and add a percent sign.

#### Changing a Percent to a Decimal
Express 130% as a decimal

To express a percent as a decimal, divide by 100 to move the decimal point two places to the left, and remove the percent sign.

#### Finding the Total Price Including Tax
San Antonio's current sales tax rate is 8.25%. If a store is selling a graphing calculator for $109.99, how much tax is paid? What is the graphing calculator's total cost?

Sales Tax Amount = Tax Rate \( \times \) Original Price

Total Price = Original Price + Sales Tax Amount

<table>
<thead>
<tr>
<th>Sales Tax Amount</th>
<th>$9.07</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Price</td>
<td>$119.06</td>
</tr>
</tbody>
</table>

#### Finding the Total Discounted Price
Hand sanitizer that normally sells for $5.79 is now on sale at 40% off. What is the hand sanitizer's discounted price?

Discount Amount = Discount Rate \( \times \) Original Price

Discounted Price = Original Price - Discount Amount

<table>
<thead>
<tr>
<th>Discount Amount</th>
<th>$2.32</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discounted Price</td>
<td>$3.47</td>
</tr>
</tbody>
</table>

#### Showing Percent Increase/Decrease
An 8GB Reading Tablet normally sells for $139.00. The discounted price is $119.00. What is the percent decrease of the tablet?

Percent Change = \( \frac{\text{Original Amount} - \text{New Amount}}{\text{Original Amount}} \)

Positive Percent Change is a Percent Decrease
Negative Percent Change is a Percent Increase

<table>
<thead>
<tr>
<th>Percent Change</th>
<th>$20.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent Change</td>
<td>$139.00</td>
</tr>
<tr>
<td>Percent Change</td>
<td>.144</td>
</tr>
<tr>
<td>Percent Change</td>
<td>14.4%</td>
</tr>
</tbody>
</table>

#### Calculating Simple Interest
You deposit $1500 in a savings account. Your bank provides a 0.01% rate for savings accounts. Find the interest earned after 1 year.

\[ I = PV \cdot r \cdot t \]

<table>
<thead>
<tr>
<th>I</th>
<th>$15.00</th>
</tr>
</thead>
</table>

#### Using the Future Value with Simple Interest Formula
You decide to buy a certificate of deposit (CD) from your local bank. The two year CD offers a rate of 0.20%. How much must you put in the CD to have $3000 in two years?

FV = PV \times (1 + r \cdot t)

<table>
<thead>
<tr>
<th>FV</th>
<th>$3000</th>
</tr>
</thead>
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</tr>
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<td>FV</td>
<td>$3000</td>
</tr>
<tr>
<td>FV</td>
<td>$2988.05</td>
</tr>
<tr>
<td>PV</td>
<td>$2988.05</td>
</tr>
</tbody>
</table>

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\( I \): Interest; \( PV \): Present Value; \( r \): Annual Interest Rate; \( t \): Time (years); \( FV \): Future Value; \( m \): Compounding Periods (per year); \( PMT \): Payment/Deposit
### Using the Future Value with Compound Interest Formula (Compounded m Times per Year)

You deposit $1500 in a savings account. Your bank provides a 0.01% annual rate for savings accounts. The interest is compounded monthly. Find the value of the account after one year.

\[
FV = PV \left( 1 + \frac{r}{m} \right)^{m \cdot t}
\]

\[
FV = PV(1 + i)^n
\]

\[
\begin{align*}
FV &= 1500 \left( 1 + \frac{0.01}{12} \right)^{12} \\
&= 1500 \left( 1.00008333 \right)^{12} \\
&= 1500 \cdot 1.00010132 \\
&= 1500.15
\end{align*}
\]

### Using the Future Value with Compound Interest Formula (Compounded Continuously)

Parents wish to have $80,000 available for their child’s education. If their child is three years old, how much must they set aside at 5% compounded continuously to meet their goal when the child is 18?

\[
FV = PV \cdot e^{r \cdot t}
\]

\[
\begin{align*}
\$80,000 &= PV \cdot e^{0.05 \cdot (18 - 3)} \\
\$80,000 &= PV \cdot e^{0.05 \cdot 15} \\
\$37,789.33 &= PV
\end{align*}
\]

### Comparing Different Interest Compounding Periods (Using the Annual Percentage Yield [APY])

What is the APY of an account paying 8% compounded quarterly?

\[
APY = \left( 1 + \frac{r}{m} \right)^m - 1
\]

\[
APY = \left( 1 + \frac{0.08}{4} \right)^4 - 1 = 0.084446 \\
APY = 8.4446% \\
\]

### Calculating the Future Value of an Annuity (Present Payments)

Suppose when you are 30, you decide to save for retirement by depositing $4,000 into a Roth IRA at the end of each year. If the interest rate is 6.5% compounded annually, how much will you have from the Roth IRA after 35 years? How much is from interest?

\[
FV = \frac{PMT \cdot \left( 1 + \frac{r}{m} \right)^{m \cdot t} - 1}{\frac{r}{m}}
\]

\[
\begin{align*}
FV &= \frac{4000 \cdot \left( 1 + \frac{0.065}{1} \right)^{1 \cdot 35} - 1}{\frac{0.065}{1}} \\
FV &= \frac{4000 \cdot \left( 1.065 \right)^{35} - 1}{0.065} \\
FV &= \frac{4000 \cdot 8.10616851 - 1}{0.065} \\
FV &= \frac{32,425.67304 - 1}{0.065} \\
FV &= \frac{32,424.67304}{0.065} \\
FV &= 496,376.66
\end{align*}
\]

\[
l = 496,376.66 - 4000(35) \\
l = 356,376.66
\]

### Calculating a Sinking Fund Payment

Parents wish to have $80,000 available for their child’s education. If their child is three years old, how much should they deposit per month at 5% compounded monthly to meet their goal when the child is 18?

\[
PMT = \frac{FV \cdot \frac{r}{m}}{\left[ 1 + \frac{r}{m} \right]^{m \cdot t} - 1}
\]

\[
\begin{align*}
PMT &= \frac{80,000 \cdot \frac{0.05}{12}}{\left[ 1 + \frac{0.05}{12} \right]^{12 \cdot 15} - 1} \\
PMT &= \frac{80,000 \cdot \frac{0.05}{12}}{\left( 1 + \frac{0.05}{12} \right)^{180} - 1} \\
PMT &= \frac{80,000 \cdot 0.00416667}{\left( 1.00416667 \right)^{180} - 1} \\
PMT &= \frac{333.33}{\left( 1.00416667 \right)^{180} - 1} \\
PMT &= \frac{333.33}{0.84870903} \\
PMT &= 392.77
\end{align*}
\]

### Calculating the Present Value of an Annuity (Future Withdraws)

How much should you deposit in an account paying 8% compounded weekly in order to receive weekly payments of $150 for the next 3 years?

\[
PV = \frac{PMT \cdot \left[ 1 - \left( 1 + \frac{r}{m} \right)^{-m \cdot t} \right]}{\frac{r}{m}}
\]

\[
\begin{align*}
PV &= \frac{150 \cdot \left[ 1 - \left( 1 + \frac{0.08}{52} \right)^{-52 \cdot 3} \right]}{\frac{0.08}{52}} \\
PV &= \frac{150 \cdot \left[ 1 - \left( 1 + 0.00096154 \right)^{-156} \right]}{0.00153846} \\
PV &= \frac{150 \cdot 0.94870413}{0.00153846} \\
PV &= \frac{142.28}{0.00153846} \\
PV &= 92,419.07
\end{align*}
\]

### Calculating Fixed Installment Loan Payment (Amortization)

Suppose you decide to borrow $30,000 for a new car. If you accept a 5 year loan at 4%, what are the monthly payments? What is the total interest?

\[
PMT = \frac{PV \cdot \frac{r}{m}}{\left[ 1 - \left( 1 + \frac{r}{m} \right)^{-m \cdot t} \right]}
\]

\[
\begin{align*}
PMT &= \frac{30,000 \cdot \frac{0.04}{12}}{\left[ 1 - \left( 1 + \frac{0.04}{12} \right)^{-12 \cdot 5} \right]} \\
PMT &= \frac{30,000 \cdot 0.00333333}{\left( 1 + 0.00333333 \right)^{60} - 1} \\
PMT &= \frac{100}{\left( 1.00333333 \right)^{60} - 1} \\
PMT &= \frac{100}{0.27322787} \\
PMT &= 366.37
\end{align*}
\]

\[
l = 366.37(60) - 30,000 \\
l = 21,982
\]

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<table>
<thead>
<tr>
<th>Interest</th>
<th>PV: Present Value</th>
<th>r: Annual Interest Rate</th>
<th>t: Time (years)</th>
<th>FV: Future Value</th>
<th>m: Compounding Periods (per year)</th>
<th>PMT: Payment/Deposit</th>
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